# Applecross Senior High School

### Semester One Examination, 2019

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNIT 1**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

(a) The points and have coordinates and respectively. If is the midpoint of and , determine the coordinates of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes equations for midpoints   -coordinate of   -coordinate of |

(b) The points and have coordinates and respectively, where and are constants. Determine the value of and the value of if the midpoint of and is at . (3 marks)

|  |
| --- |
| **Solution** |
| Solve simultaneously CAS to get |
| **Specific behaviours** |
| ✓ equations for both coordinates of midpoint   value of   value of |

Question 10 (8 marks)

(a) The variables and are directly proportional and when .

(i) Determine an equation for the relationship between and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of line through origin   correct relationship |

(ii) State the value of when . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(b) The time, minutes, that a car takes to travel one kilometre at a constant speed of kmh-1 is given by the formula .

(i) Determine the value of the constant , given that when . (1 mark)

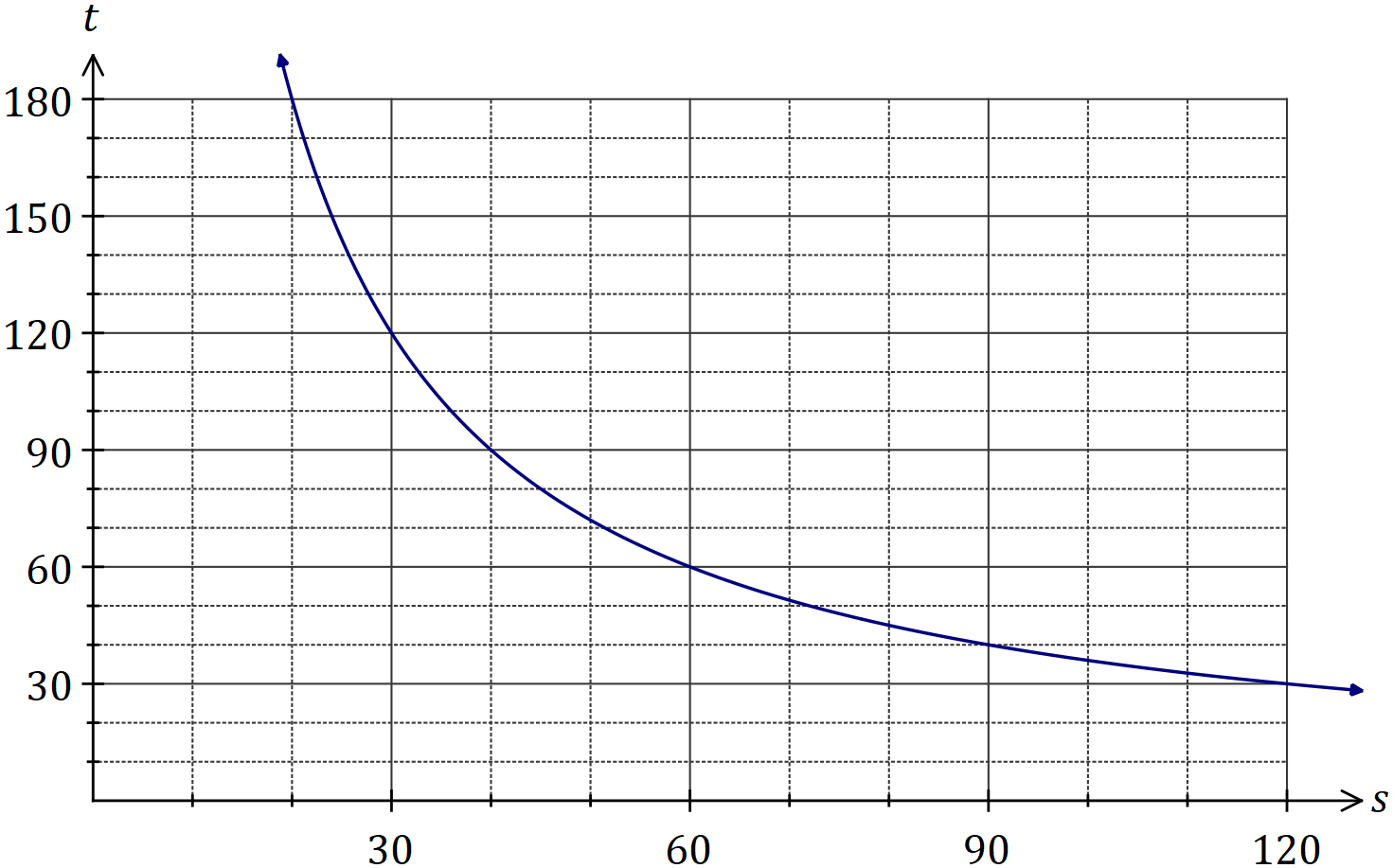
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) Determine the value of when . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(iii) On the axes below, draw a graph to show how varies with . (3 marks)

|  |
| --- |
| **Solution** |
| See graph. Key points: |
| **Specific behaviours** |
| ✓ plots at least 3 key points   plots all key points   smooth curve |



Question 11 (8 marks)

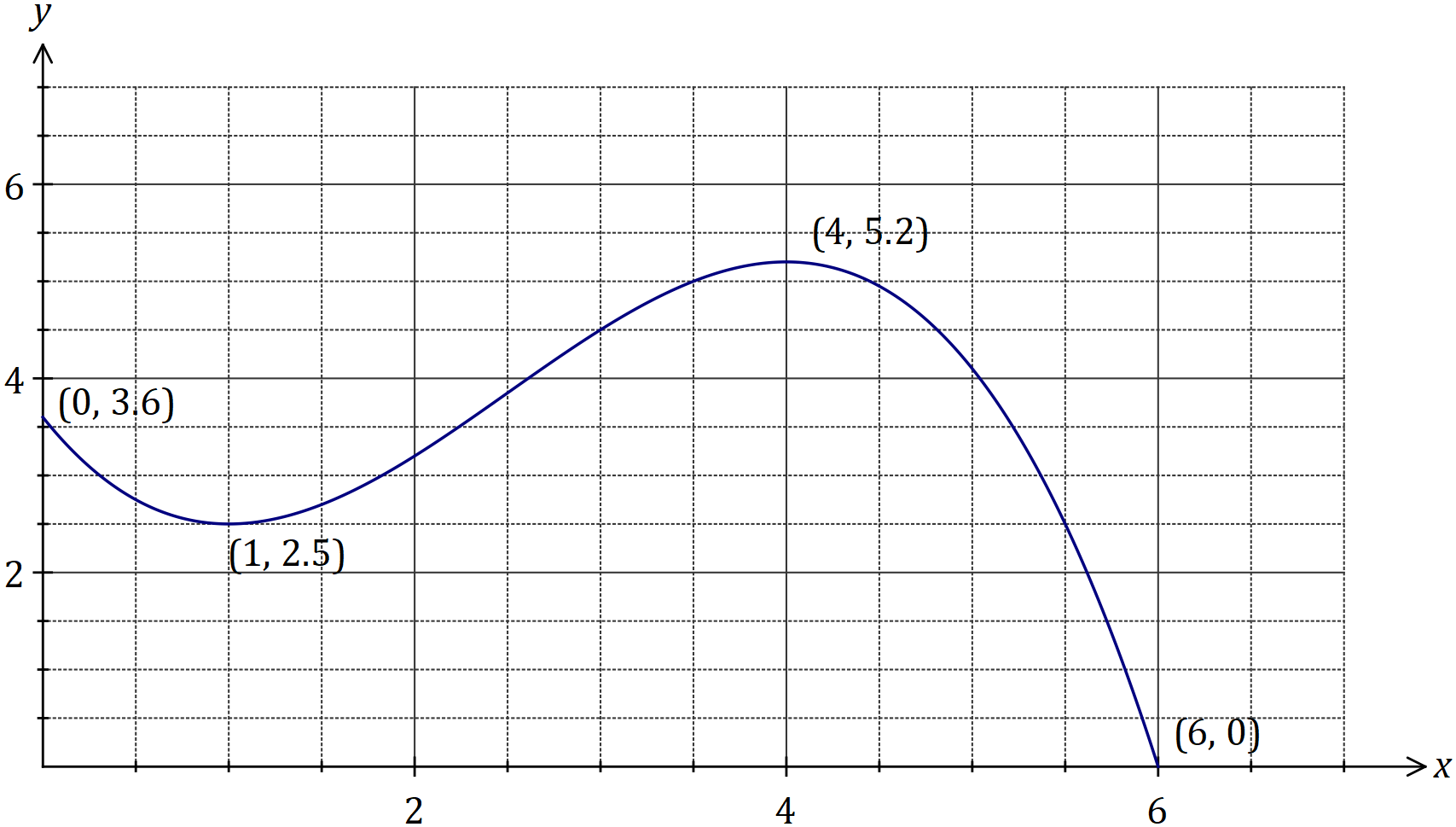
In an experiment, the sound intensity, , can be modelled by , where is the distance from the sound source in metres and .

(a) Determine when . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(b) Draw the graph of on the axes below. (4 marks)

|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ intercepts   accurate maximum   accurate minimum   smooth curve |



(c) Determine the equation of the straight line that passes through the -intercept and the -intercept of the graph of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ gradient   -intercept and equation |

(d) Determine the coordinates of the point of intersection of with the graph of where and . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct coordinates |

Question 12 (8 marks)

(a) A function is given by .

(i) State the natural domain and corresponding range of . (2 marks)

|  |
| --- |
| **Solution** |
| Domain:  Range: |
| **Specific behaviours** |
| ✓ Domain   Range |

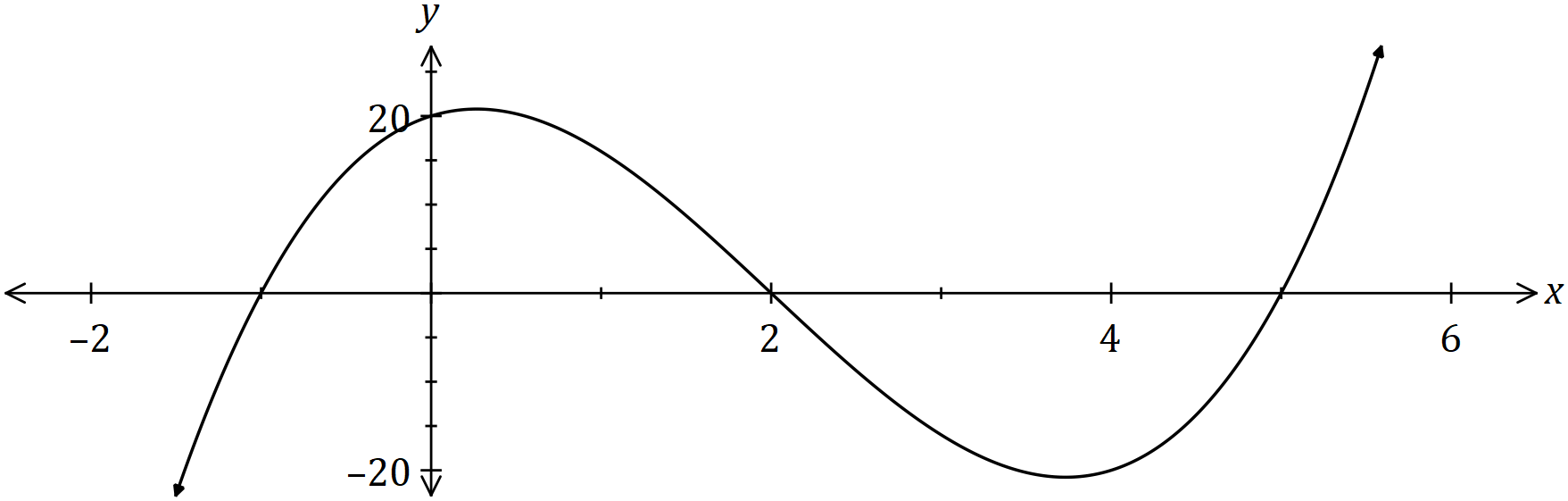
(ii) The graph of  is dilated vertically by a scale factor of 2. Determine the coordinates of the -axis intercept. (2 marks)

|  |
| --- |
| **Solution** |
| Y intercept is |
| **Specific behaviours** |
| ✓ ✓ correct coordinates |

(iii) The graph of  is translated 3 units to the right. Determine the equation of the translated function. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct equation |

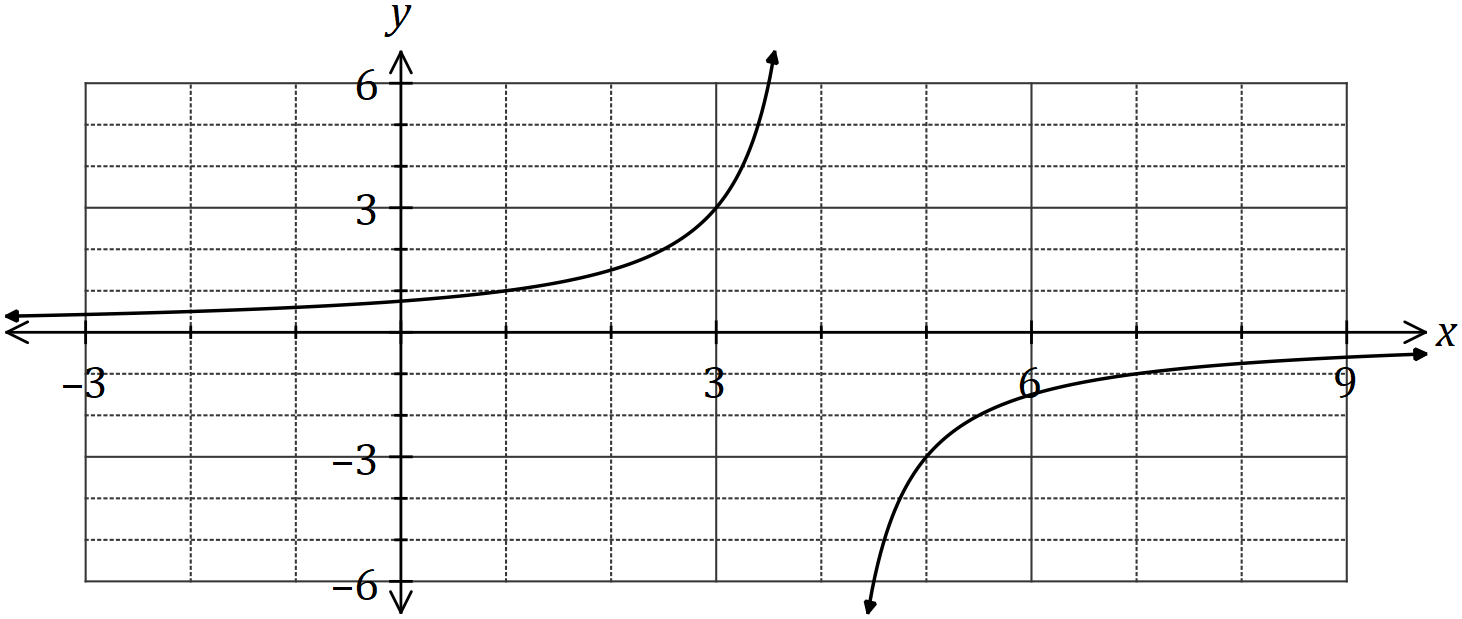
(b) The graph of the cubic function is shown below. Determine . (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ cubic in factored form   correct value of   required value |

Question 13 (8 marks)

The graph of is shown below where .



(a) State the value of the constant and the value of the constant . (3 marks)

|  |
| --- |
| **Solution** |
| Using |
| **Specific behaviours** |
| ✓ value of   uses point on curve   value of |

(b) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vertical asymptote   horizontal asymptote |

(c) Describe the transformation required to transform the graph of to obtain the graph of and state the domain and range of the transformed function. (3 marks)

|  |
| --- |
| **Solution** |
| Translate the graph 3 unit to the right.  Domain:  Range: |
| **Specific behaviours** |
| ✓ transformation   domain   range |

Question 14 (8 marks)

(a) Convert, giving an exact answer

(i) to radians. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  exact value |

(ii) radians to degrees. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  exact value |

(b) Calculate, to the nearest degree, the acute angle between the line and the line . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of   one correct angle with -axis   correct angle between lines |

(c) The sides adjacent to the right-angle in a right triangle have lengths cm and cm.

If the smallest angle in the triangle is , determine an exact value for

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct ratio |

(i) . (1 mark)

(ii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates length of hypotenuse   correct ratio for small |

Question 15 (7 marks)

An **obtuse** angled triangle has cm, cm and an area of cm2, given the length of b is the longest length of a side of triangle ABC.

(a) Sketch a triangle to show this information. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ triangle with all information |

(b) Determine the size of . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes into area equation   correct (obtuse) angle |

(c) Show that cm. (2 marks)

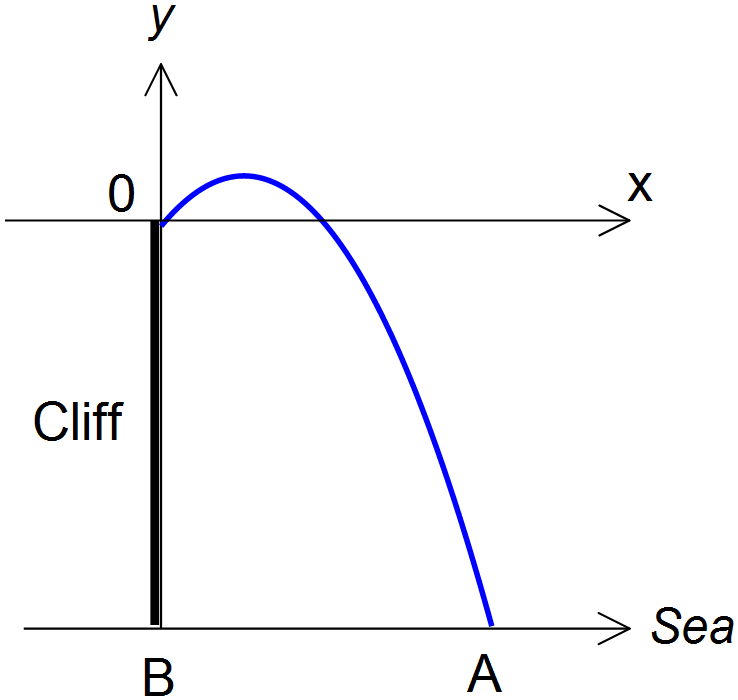
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses appropriate equation that includes   substitutes correctly and solves to at least 1 dp |

(d) Show that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses appropriate equation that includes   substitutes correctly and solves to at least 1 dp |

Question 16 (8 marks)

A ball is thrown off the top of a cliff, 100m above sea level. Taking the point of projection O as the origin of the coordinate axes, the path taken by the ball is given as



The ball hits the surface of the sea at A.

1. Find the height above sea level for the highest point reached by the ball. (2 marks)

|  |
| --- |
| **Solution** |
| Maximum turning point (15, 22.5)  Highest point above sea level is 100 + 22.5 = 122.5 m |
| **Specific behaviours** |
| ✓ finds the turning point   correct distance |

1. Find the distance from A to B, the base of the cliff. (3 marks)

|  |
| --- |
| **Solution** |
| A: y = -100 x = 50  B: y = -100 x = 0  Distance AB = 50 m |
| **Specific behaviours** |
| ✓ ✓ finds correct x coordinate for points A and B   correct distance AB |

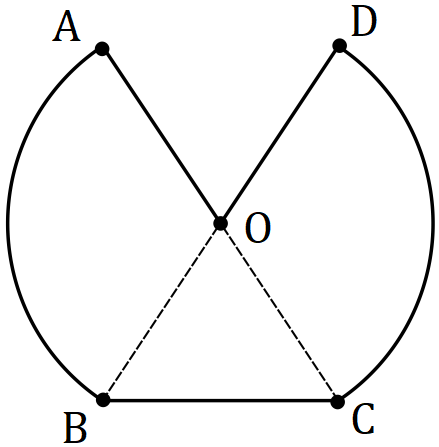
1. Find the horizontal distance from O, correct to one decimal place, when the ball is 110m

above sea level. (3 marks)

|  |
| --- |
| **Solution** |
| Y = 10 ⇒ x = 3.82, 26.18  Horizontal distance = 26.18 – 3.82  = 22.36  = 22.4 m (correct to 1 d.p.) |
| **Specific behaviours** |
| ✓ understands 110 m above sea level is y = 10   correct x values   correct distance |

Question 17 (7 marks)

In shape below, and , are diameters of the circle with centre and radius cm.



(a) Calculate the perimeter of . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  indicates length of arc   indicates use of cosine rule for   correct length   correct total perimeter |

(b) Calculate the area of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  sector area   triangle area   correct total area |

Question 18 (6 marks)

Let and .

Give your answers to the following in terms of and/or .

(a) Write down an expression for

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

(ii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

(b) Determine an expression for . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  indicates use of   expression for  ✓ correct expression |

(c) Determine an expression for . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

Question 19 (8 marks)

(a) The equation of the axis of symmetry for the graph of is . Determine the value of , using a method that does not refer to the graph of the parabola.

(2 marks)

|  |  |
| --- | --- |
| **Solution** | |
|  |  |
| **Specific behaviours** | |
| ✓ uses or partially completes the square   value of | |

(b) A parabola with equation has a turning point at and passes through the point . Determine the value of , the value of and the value of .

(3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correctly writes in turning point form   solves for using point   expands and states all values |

(c) Determine the value of the discriminant for the quadratic equation and use it to explain how many solutions the equation will have.

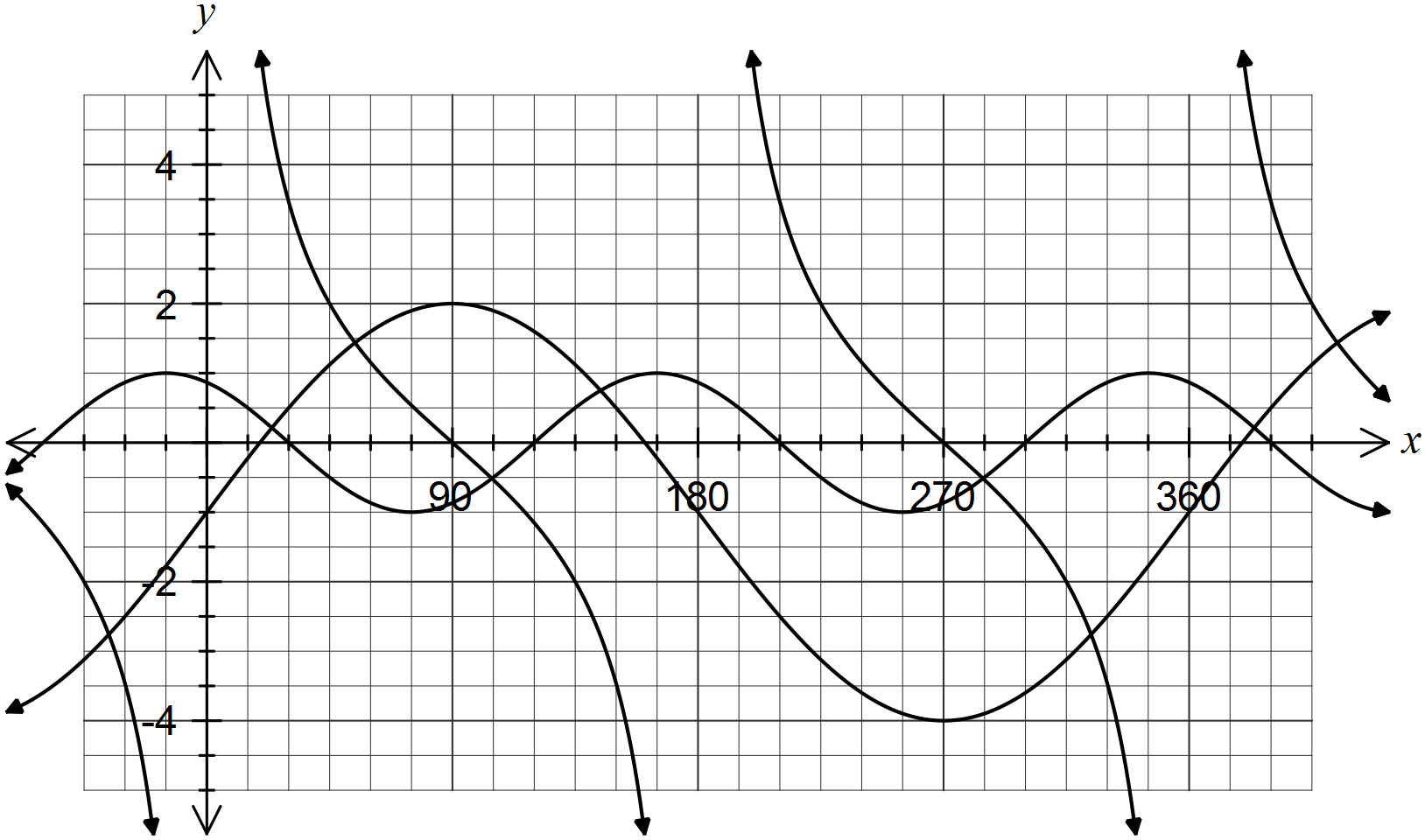
(3 marks)

|  |
| --- |
| **Solution** |
| When , quadratic will have one solution.  Hence equation will have two solutions - one from linear factor and one from quadratic factor. |
| **Specific behaviours** |
| ✓ value of discriminant   uses discriminant to say quadratic will have one solution   explains why equation has two solutions |

Question 20 (8 marks)

(a) The graphs of three functions,

are shown below. (6 marks)



Determine the values of the real constants .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  each correct number |

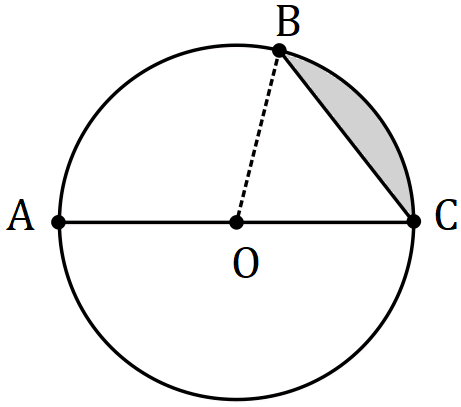
(b) Determine the equation of a line which is inclined at an angle of 120º to the positive

-axis and cuts the -axis at (0, -2). (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct gradient   correct equation |

Question 21 (8 marks)

(a) The circle shown has centre and diameter of length cm. Determine the shaded area given that . (4 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equation using angles   correct angle for segment   substitutes correctly into formula   correct area |

(b) A sector of a circle has a perimeter of cm and an area of cm2. Determine the radius of the circle. (4 marks)

|  |
| --- |
| **Solution** |
| Solving simultaneously gives  Hence |
| **Specific behaviours** |
| ✓ equation for perimeter   equation for area   solution of equations   states both values of |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

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